Problem 7

p^7

p^6

p^5

p^4

p^3

p^2

p

1

p^7q

p^7

p^6q

p^6

p^5q

p^5

p^4q

p^4

p^3q

p^3

p^2q

p^2

pq

p

q

1

		p^7q^2
	p^7q	
p^7		
		p^6q^2
	p^6q	
p^6		
	p^5q	p^5q^2
p^5		
		p^4q^2
	p^4q	
p^4		
		p^3q^2
	p^3q	
p^3		
		p^2q^2
	p^2q	
p^2		
		pq^2
	pq	
p		
		q^2
	q	
1		

Theorem 1. If the game is restricted to numbers of the form $N = p^\alpha q^\beta$ (powers of two distinct primes) then a number is losing for the player who has to divide it iff $\alpha = \beta$ mod 3. (then we say that N has the same *3-modularities*)

Proof: We must show that 1) a player who has to divide a number $p^\alpha q^\beta$ which has different 3-modularities *can* always leave a quotient with the same 3-modularities, and 2) if the number has the same 3-modularities the player *must* leave a number with different 3-modularies.,

1) is true because the prime with the larger 3-modularity can be reduced to having the smaller. (Why is it sometimes impossible to reduce the smaller to the larger?)

2) is true because any allowed division changes the modularity of exactly one of p and q.

Finally we note that $N = 1 = p^0 q^0$ has equal 3-modularities, namely, 0. ∎