≡ I'm Continuously Amazed ≡

MathπATH 2018 • Lewis & Clark College

Sam Vandervelde • Proof School • July 2, 2018

Recognizing Continuity

Does the following quantity vary in a continuous or noncontinuous manner?

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Does the following quantity vary in a continuous or noncontinuous manner?

The notes in the melody to Twinkle Twinkle Little Star.

- Continuous
- Noncontinuous

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Recognizing Continuity

Does the following quantity vary in a continuous or noncontinuous manner?

Platform numbers at a certain train station in London.

- Continuous
- Noncontinuous

Recognizing Continuity

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Platform numbers at a certain train station in London.

- Continuous
- Noncontinuous

Recognizing Continuity

Does the following quantity vary in a continuous or noncontinuous manner?

The amount of time until your first bite of lunch.

- Continuous
- Noncontinuous

Recognizing Continuity

Does the following quantity vary in a continuous or noncontinuous manner?

The amount of time until your first bite of lunch.

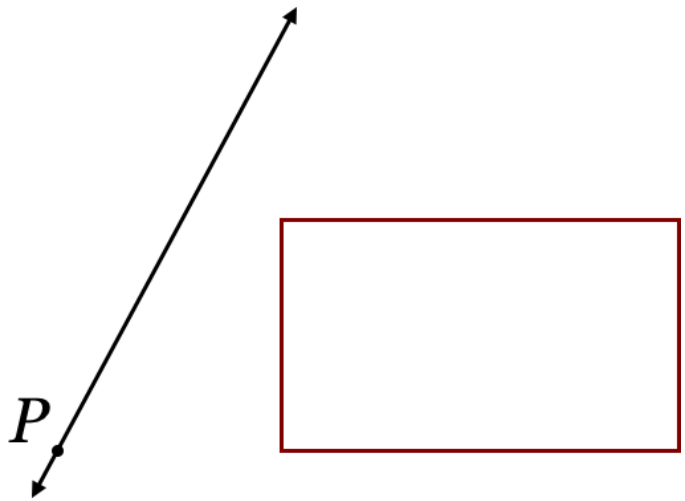
- Continuous
- Noncontinuous

Recognizing Continuity

Does the following quantity vary in a continuous or noncontinuous manner?

The fraction of the perimeter below the line as it rotates.

- Continuous
- Noncontinuous



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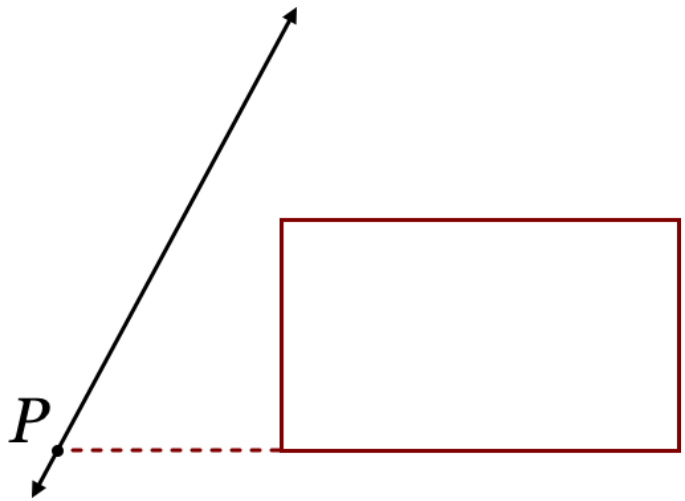

Recognizing Continuity

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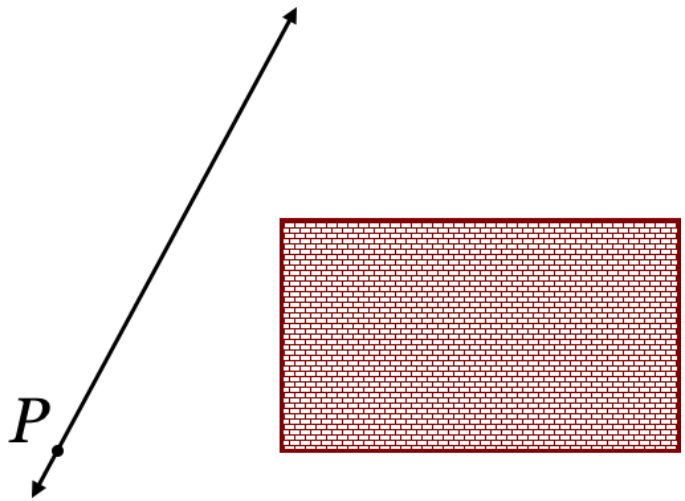
- Continuous
- Noncontinuous



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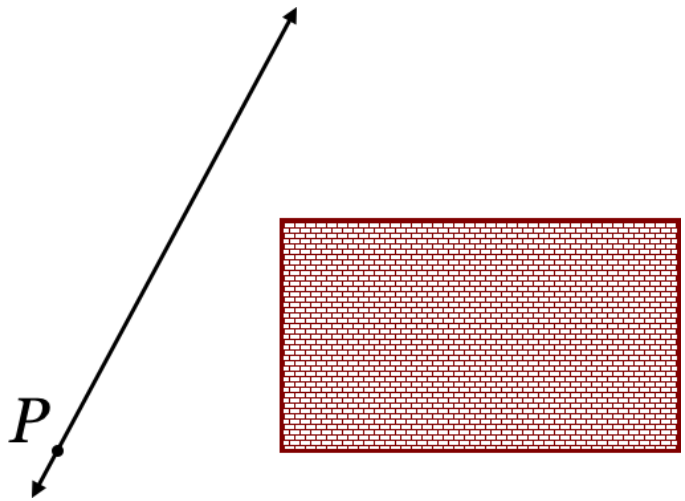
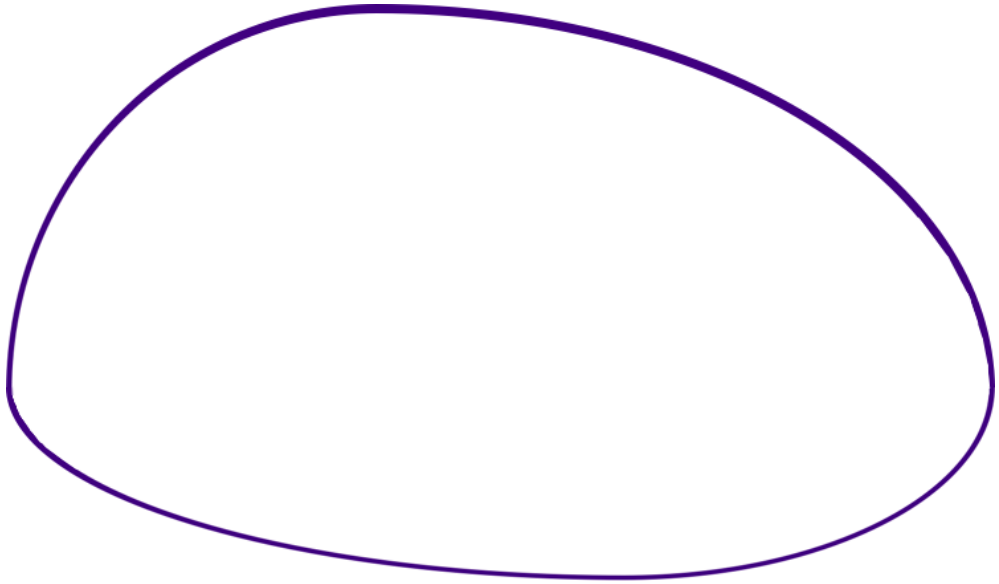


Meet the Blob

The object below is known as a **blob**. It will be the source of endless hours of fascination.



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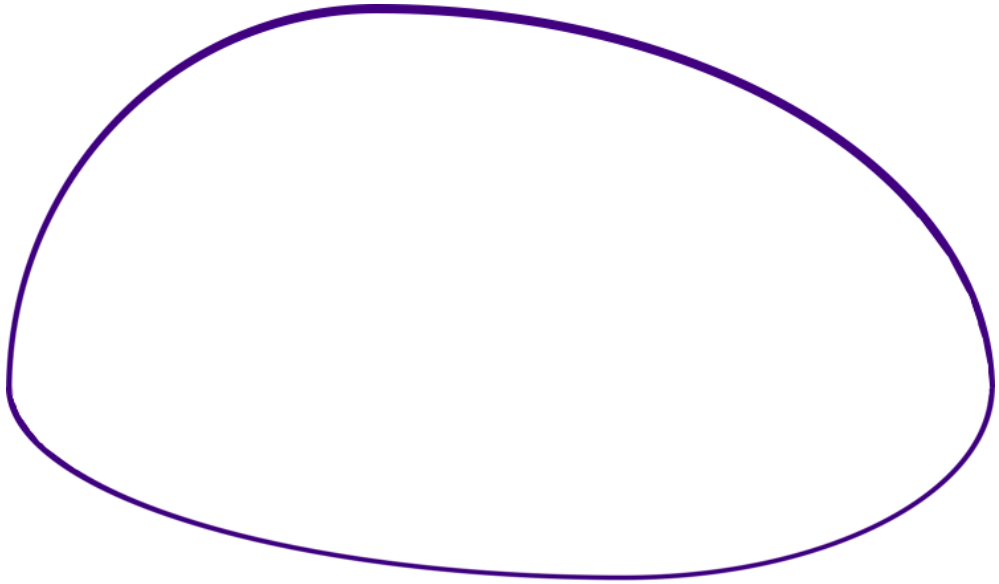
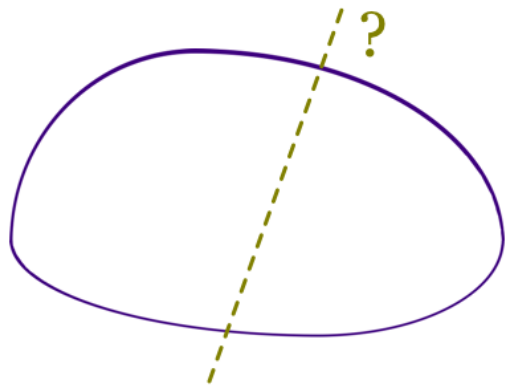


Let's characterize what makes this a blob.

Measure Your BIQ

A blob always has a line of symmetry.

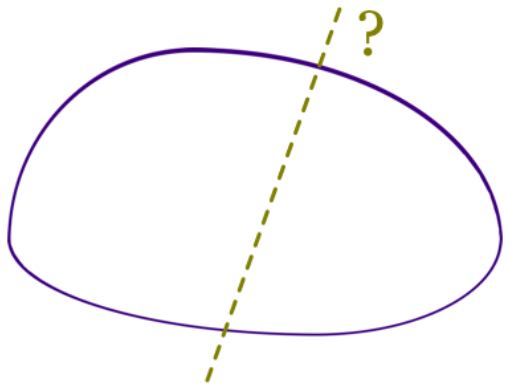
- True
- False



Blobaroni

A blob always has a line of symmetry.

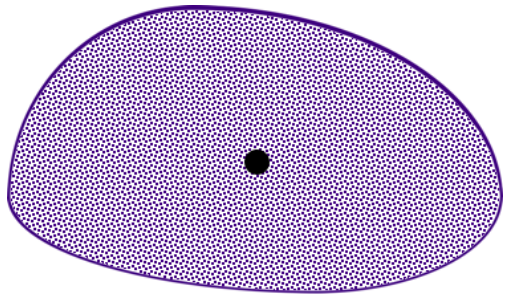
- True
- False ⟸



Measure Your BIQ

A blob always has a center of mass.

- True
- False



Blobalicious

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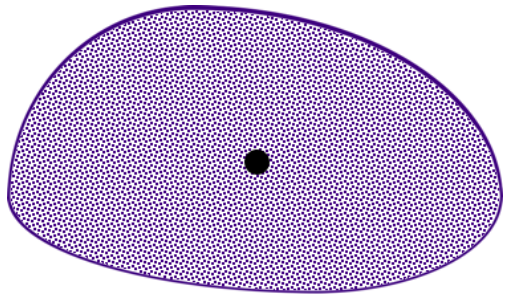
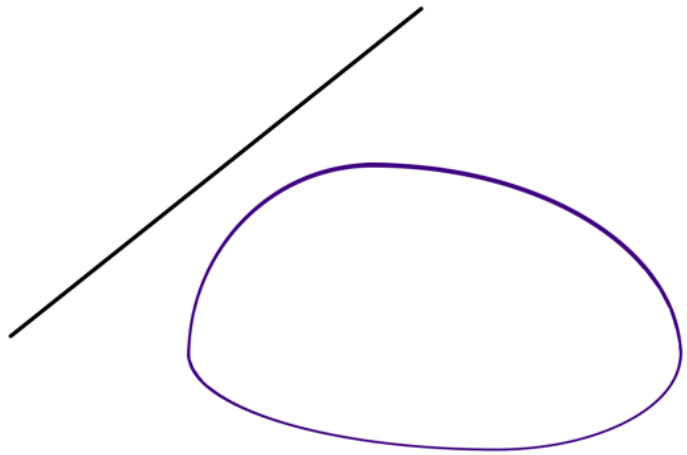


Measure Your BIQ

Given a line, there is precisely one other line parallel to it that is tangent to the blob.

- True
- False



Blob's Your Uncle

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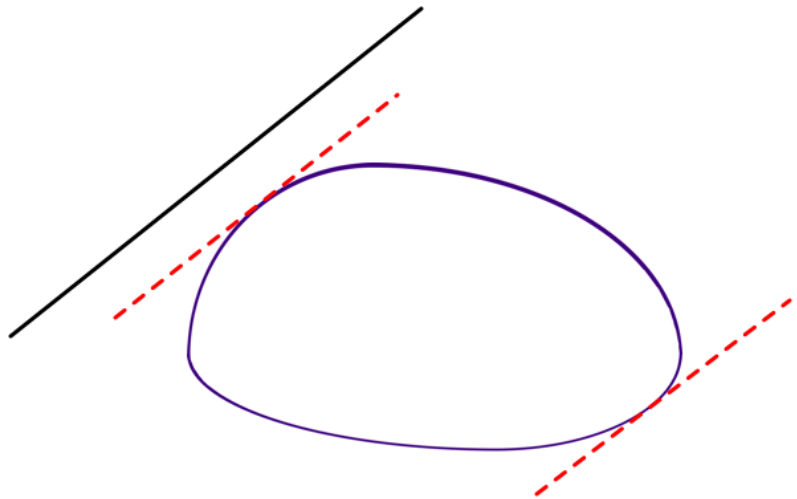
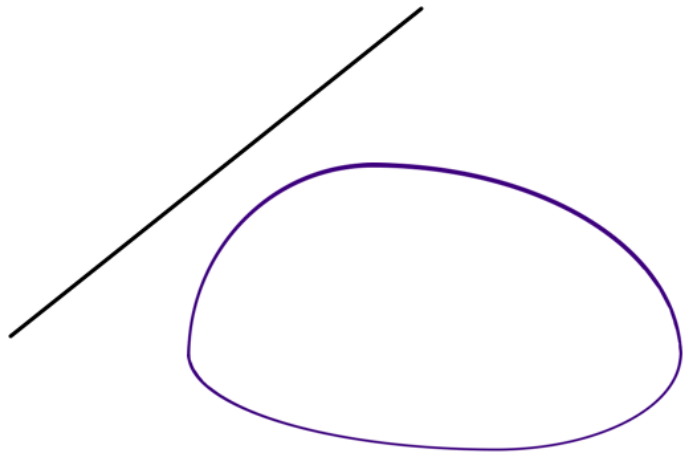


Measure Your BIQ

Given a line, there is precisely one other line parallel to it that cuts the area in half.

- True
- False



What About Blob

Given a line, there is precisely one other line parallel to it that cuts the area in half.

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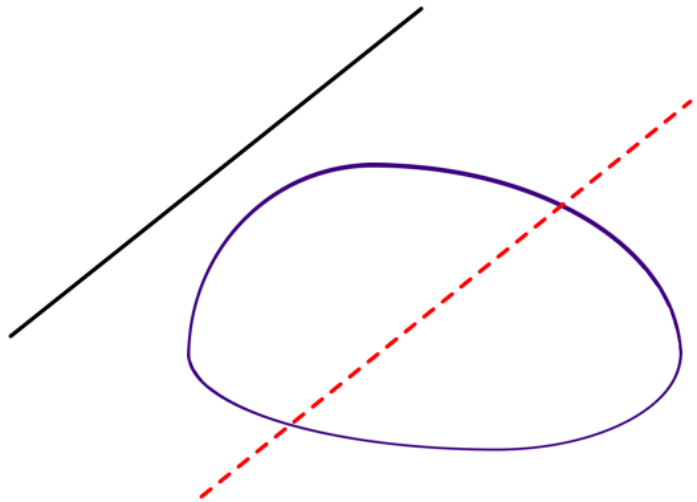


Buckle Your Shoe

To celebrate our progress, it's time for

Amazing Digit Puzzle #12

Buckle Your Shoe

To celebrate our progress, it's time for

Amazing Digit Puzzle #12

1 2

Buckle Your Shoe

To celebrate our progress, it's time for

Amazing Digit Puzzle #12

$$1\sqrt{2}$$

Buckle Your Shoe

To celebrate our progress, it's time for

Amazing Digit Puzzle #12

1.2

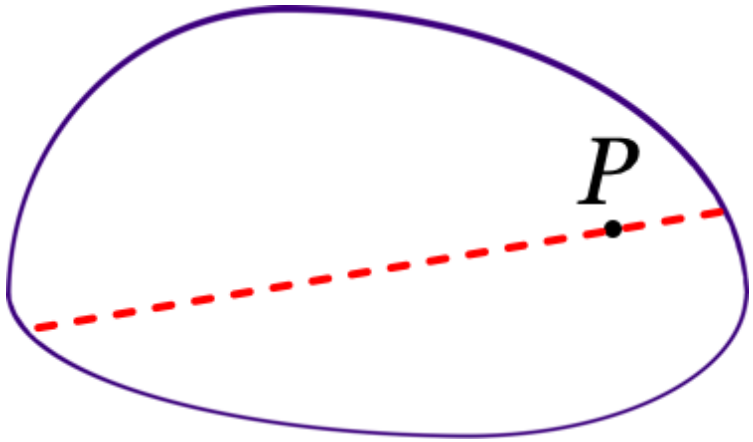
Mario Says

Things are about to get a whole lot more interesting.

Down The Middle

Given a blob with a fixed point P inside, there exists a chord whose midpoint is P.

- TRUE
- FALSE



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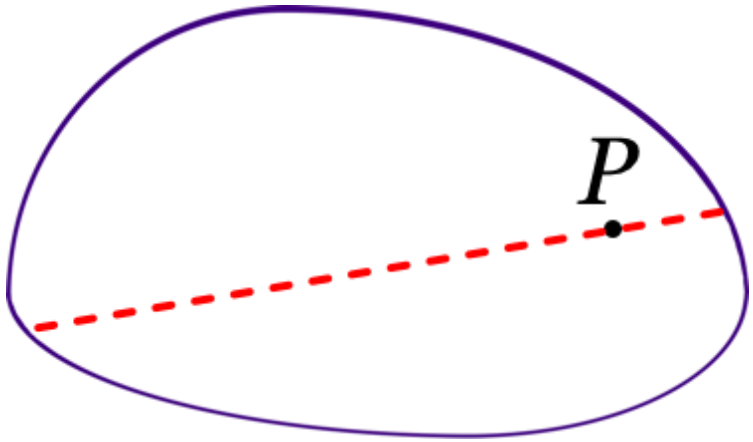

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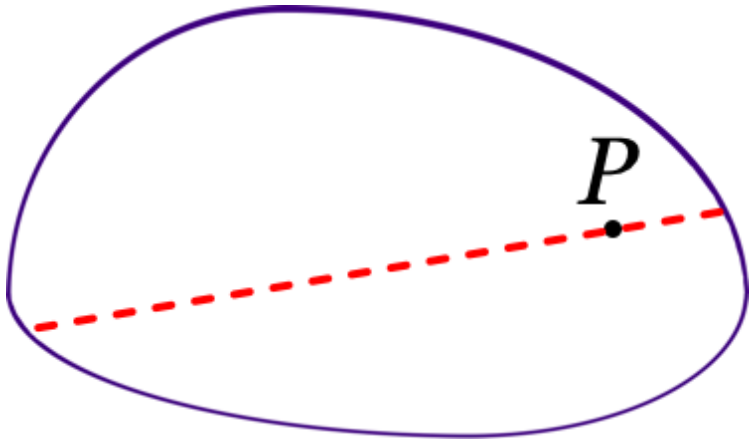
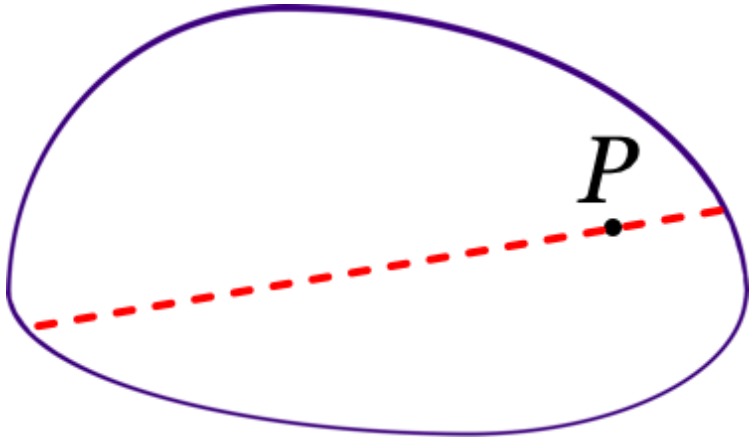


Prove it!

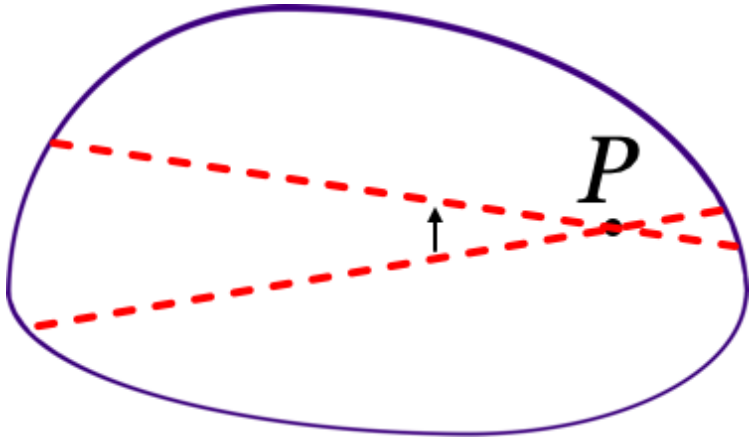
Strategy Is King

- Vary a strategic component in our diagram.



Strategy Is King

- Vary a strategic component in our diagram.
- Measure a strategic quantity that changes.



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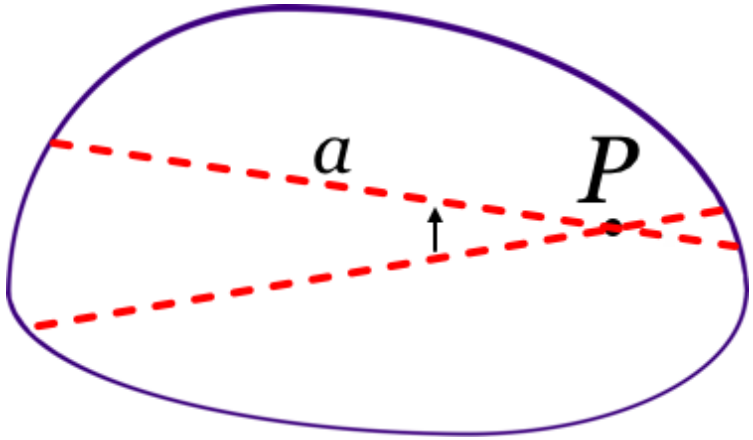

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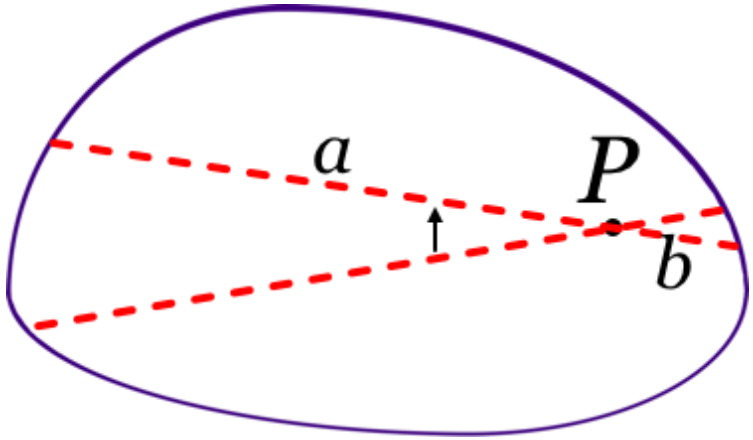
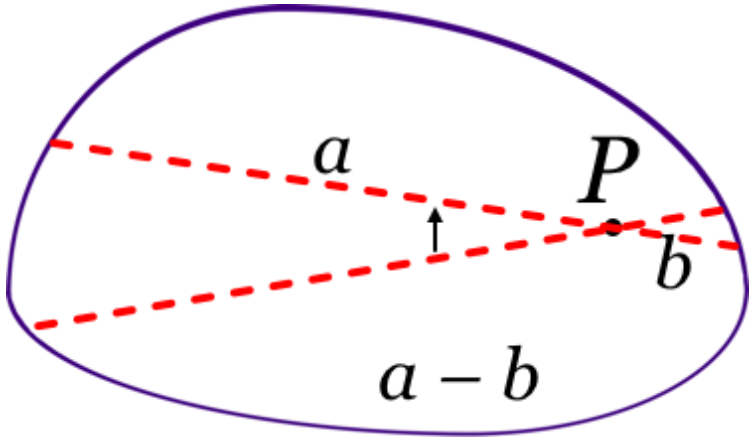


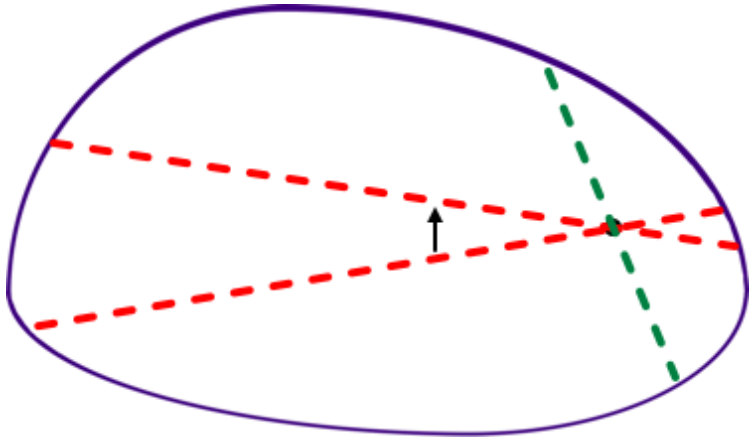
Strategy Is King

- Vary a strategic component in our diagram.
- Measure a strategic quantity that changes.
- Invoke continuity.



Strategy Is King

- Vary a strategic component in our diagram.
- Measure a strategic quantity that changes.
- Invoke continuity. *AYD*



How High Can You Go?

Bravo, everyone. Let's try out

Amazing Digit Puzzle #123

How High Can You Go?

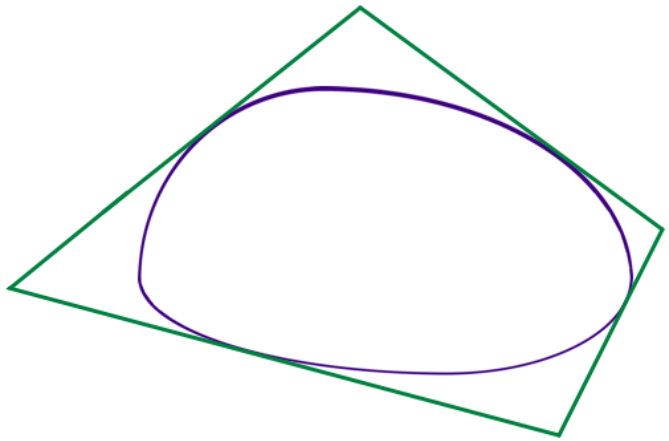
Bravo, everyone. Let's try out

Amazing Digit Puzzle #123

Packaging A Blob

Given any blob, it is always possible to circumscribe a square around it.

- True
- False



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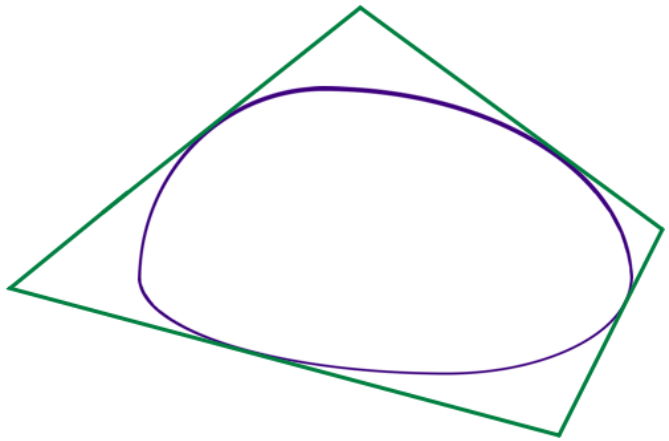

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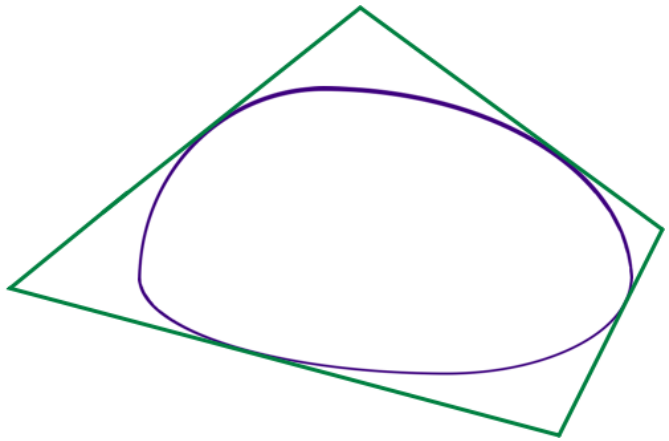
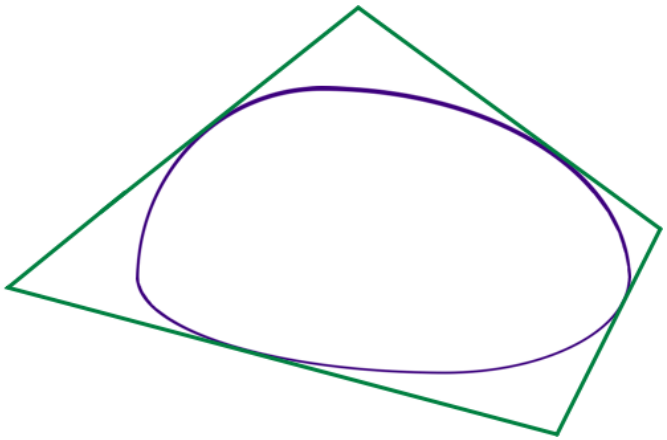


Prove it!

Strategy Is King

DISCUSS, and remember our strategy!

- Vary a component in our diagram.
- Measure a quantity that changes.
- Invoke continuity.



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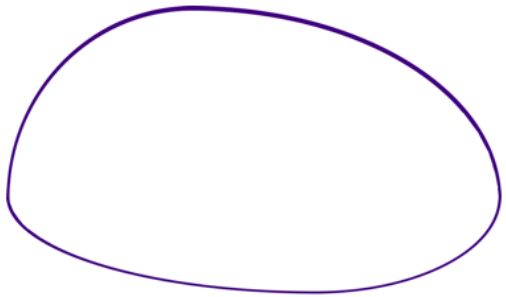

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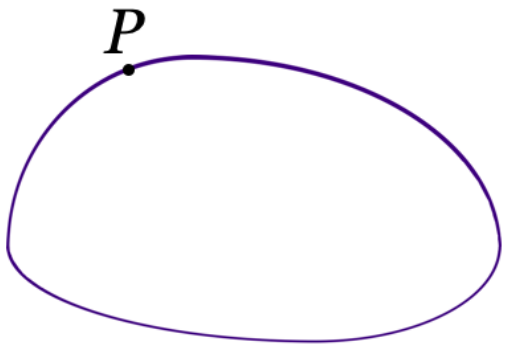

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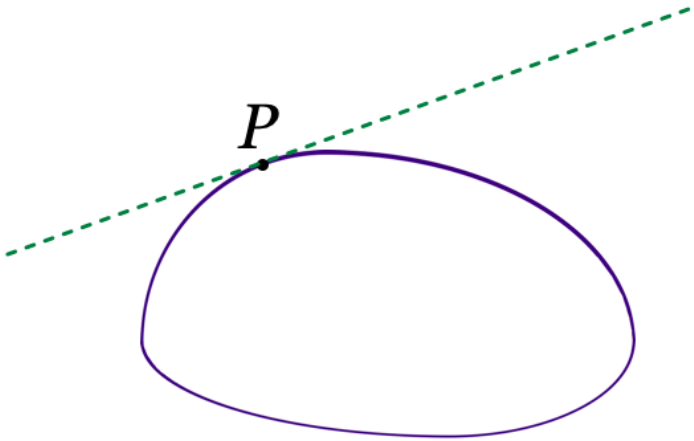

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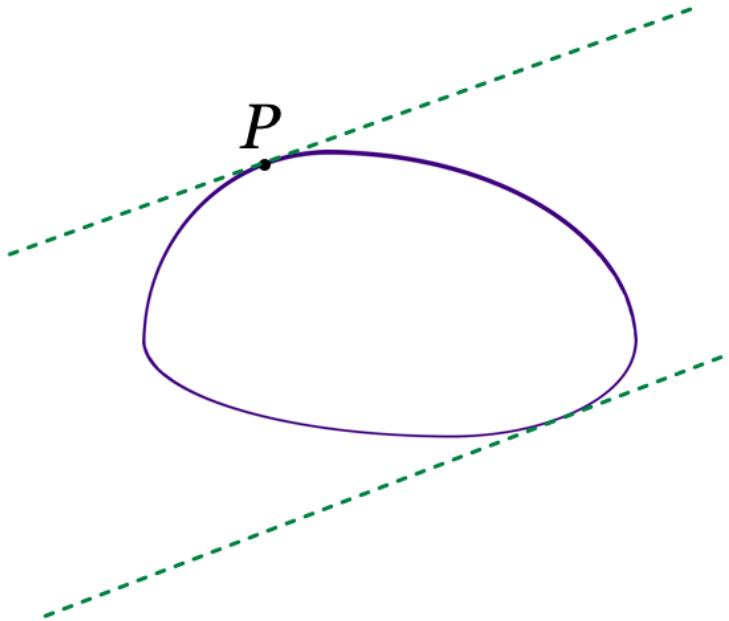

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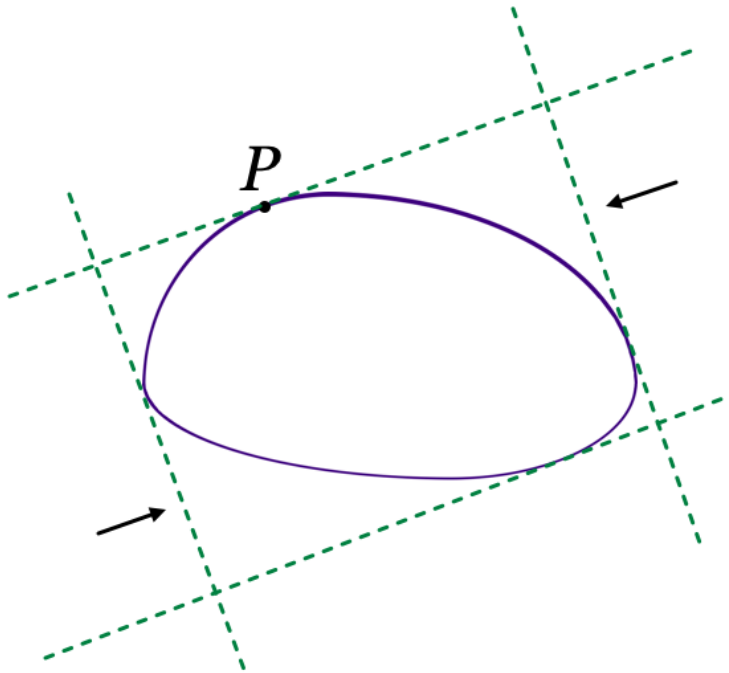

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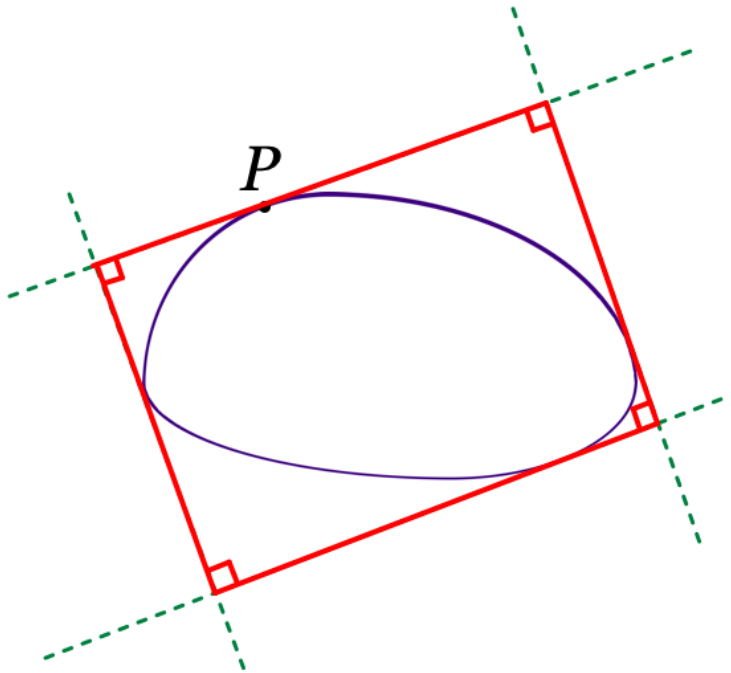

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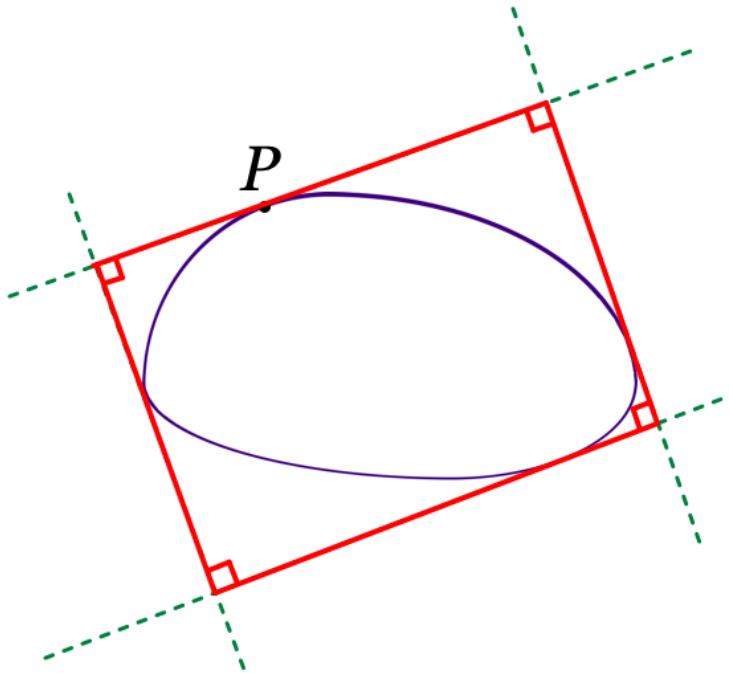
- Measure a strategic quantity that changes.



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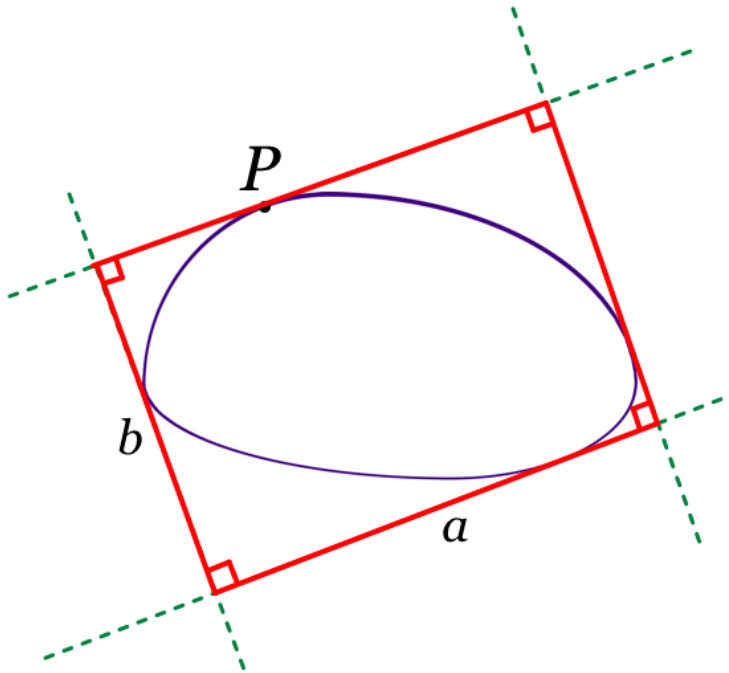

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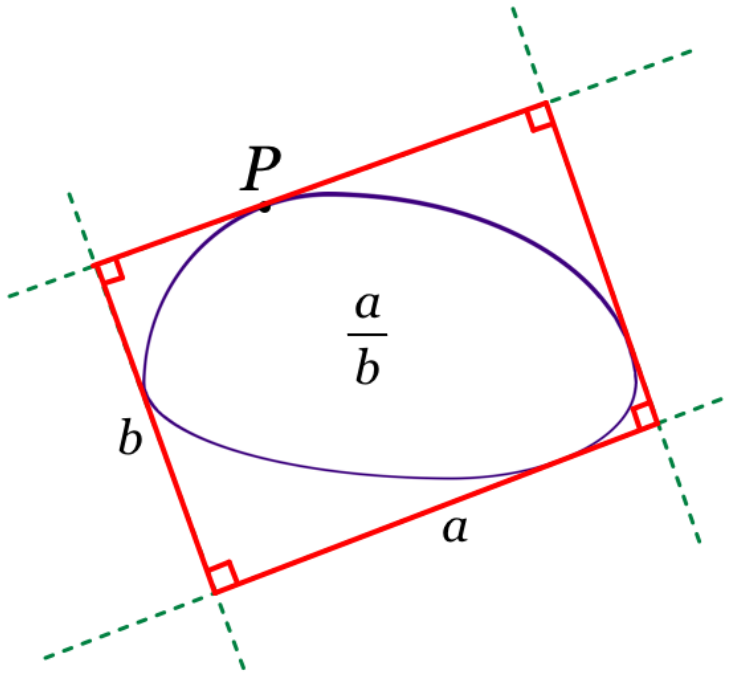

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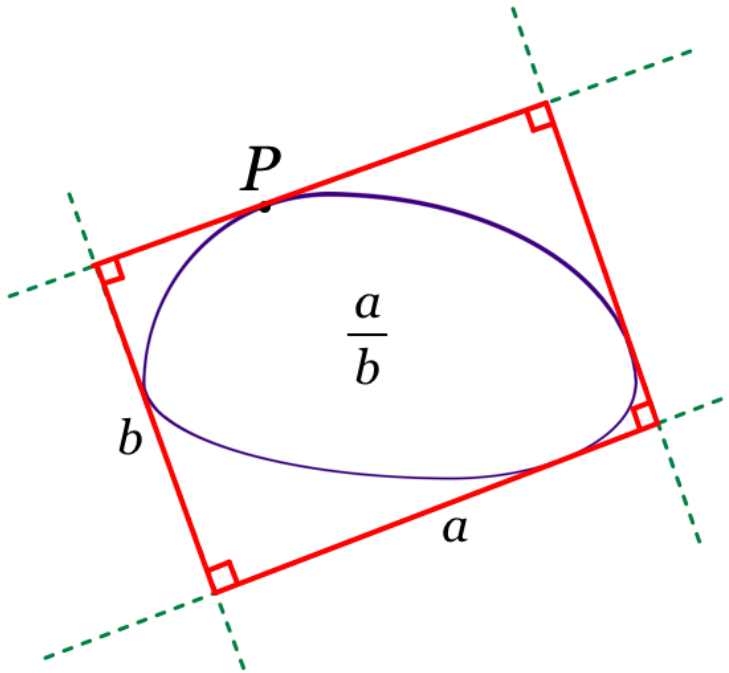
AYD!



How Low Can You Go?

Excellent job. You've earned

Amazing Digit Puzzle #1234

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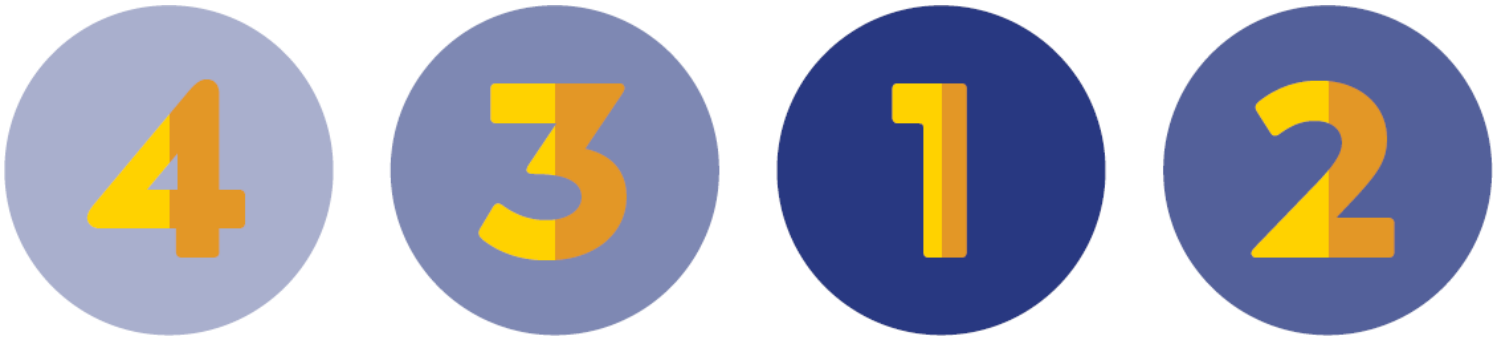
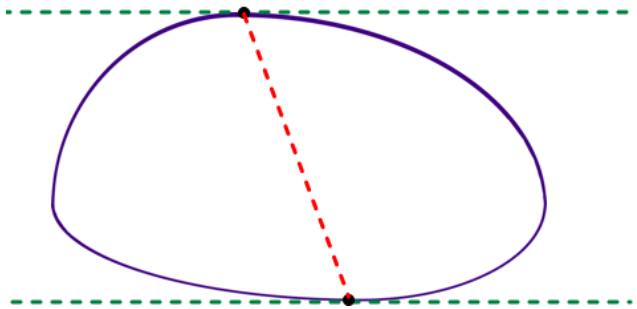
Amazing Digit Puzzle #1234



Sandwiching A Blob

Given any blob, it is always possible to squarely sandwich it between parallel lines.

- TRUE $\Longleftarrow$ (Hint)
- FALSE



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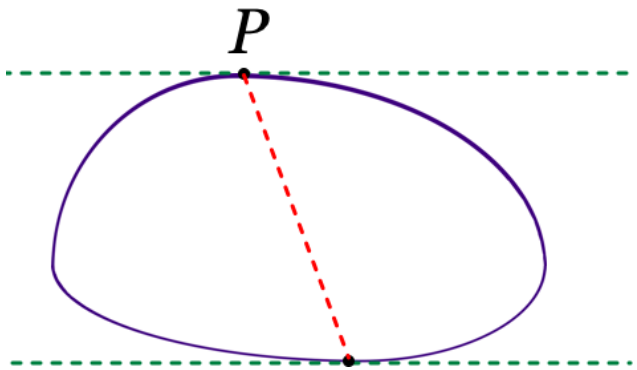

Check out this super smooth proof.

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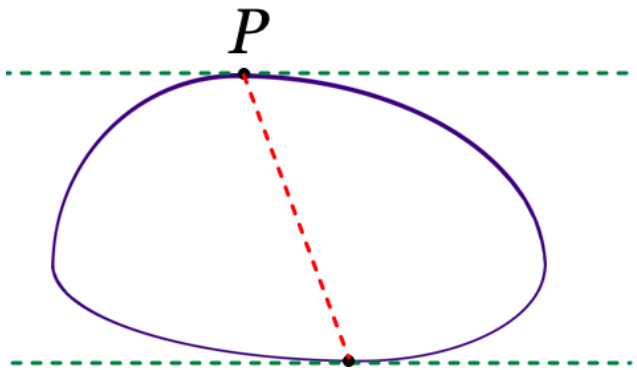

Check out this super smooth proof. That sure sounded convincing!

That Proof Was Blobous

Given any blob, it is always possible to squarely sandwich it between parallel lines.

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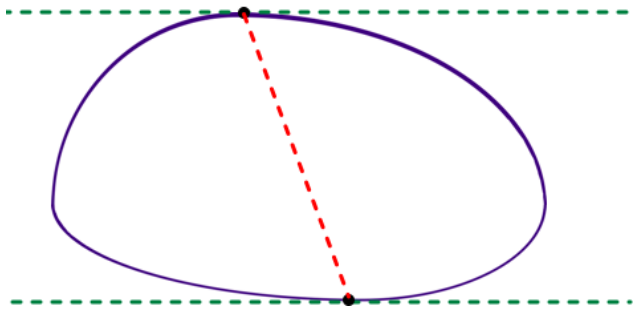

Here's a better approach: how tightly can we sandwich the blob?

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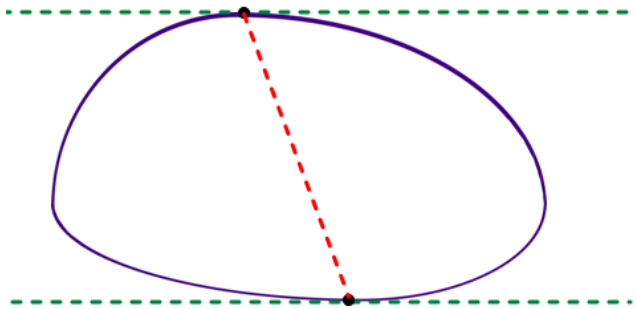


Here's a better approach: how tightly can we sandwich the blob? *AYD!*

This Will Blob Your Mind

That was delicious. Time to savor

Amazing Digit Puzzle #12345

Tip: it is a fact that $2016 = 32 \cdot 63$.

This Will Blob Your Mind

That was delicious. Time to savor

Amazing Digit Puzzle #12345

Answer #1: write $2016 = 2^5(4^3 - 1)$.

This Will Blob Your Mind

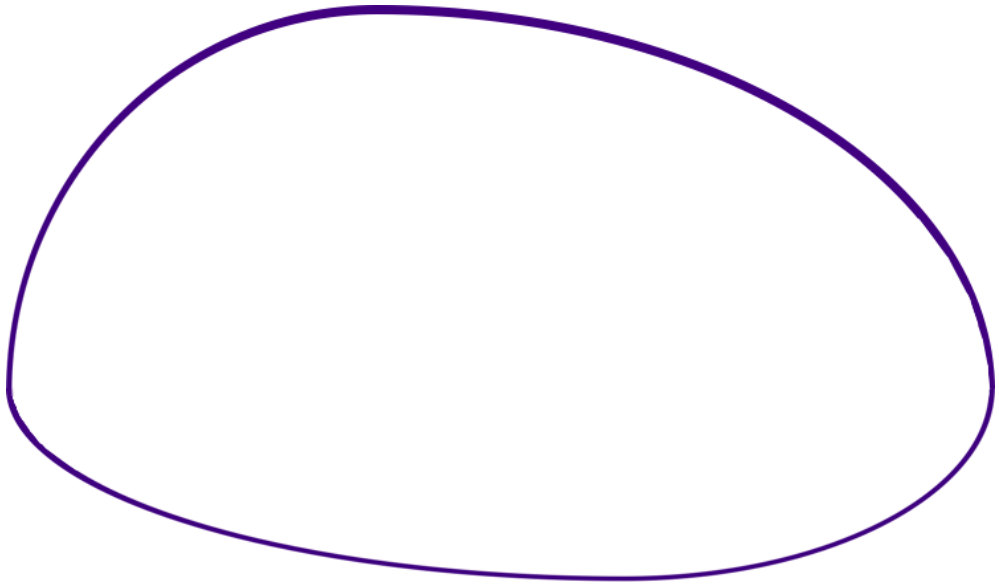
That was delicious. Time to savor

Amazing Digit Puzzle #12345

Answer #2: write $2016 = 2^4(5^3 + 1)$.

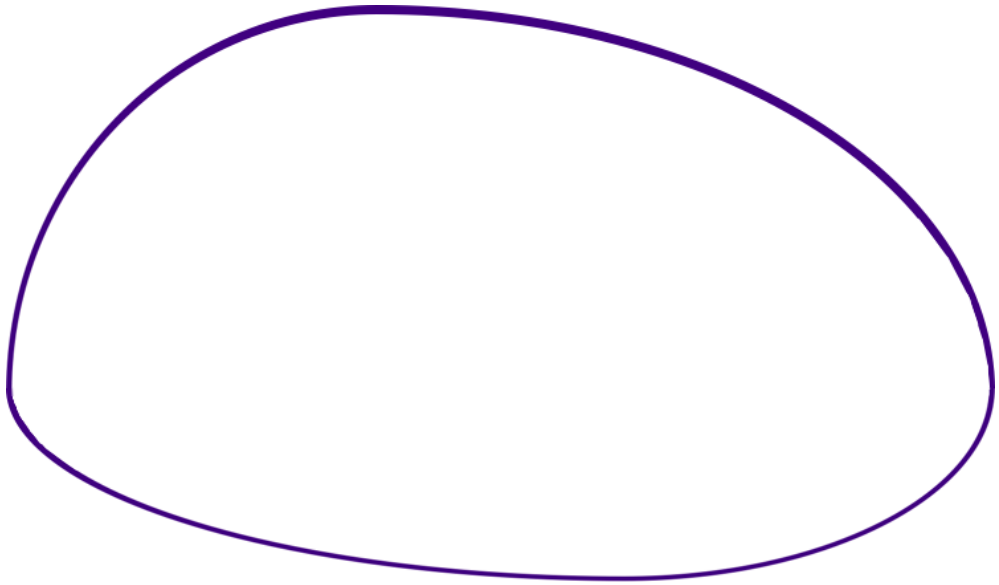
Sometimes You Feel Like A Blob

Contemplate the blob. What might exist?
Hypothesize at least two possible results.



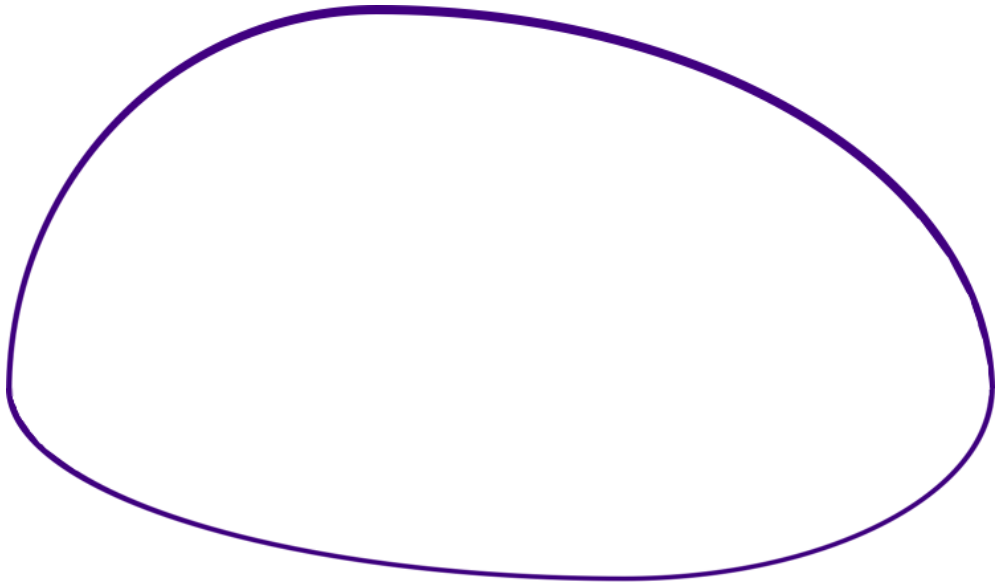
Sometimes You Feel Like A Blob

Now introduce yourself to a neighbor, and pick one of their results that intrigues you.



Sometimes You Feel Like A Blob

Finally, meet another pair of students, and select a result that is especially nice.



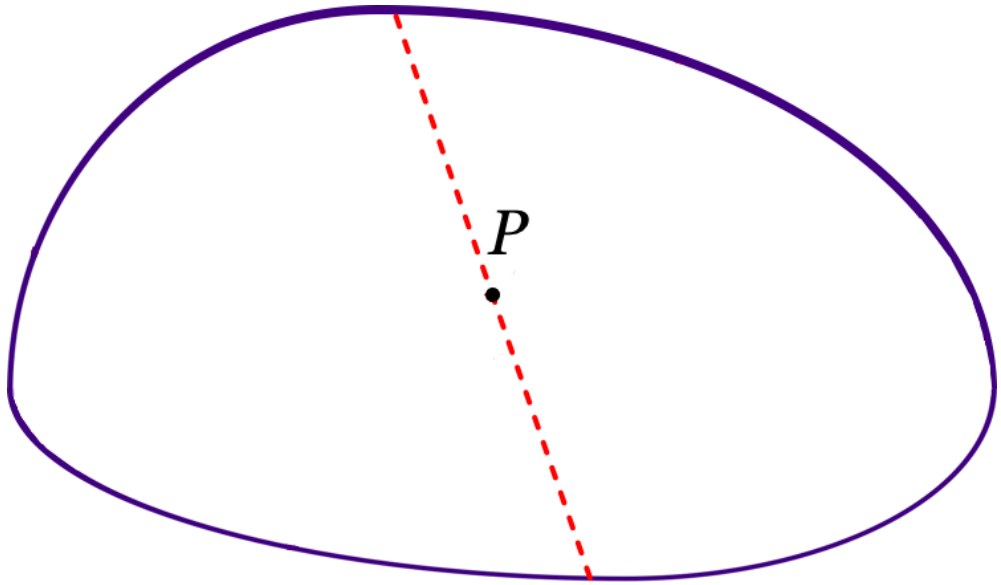
Sometimes You Feel Like A Blob

Finally, meet another pair of students, and select a result that is especially nice.

Grand Finale

Given any blob, it is always possible to locate an equiangular trisplit point.

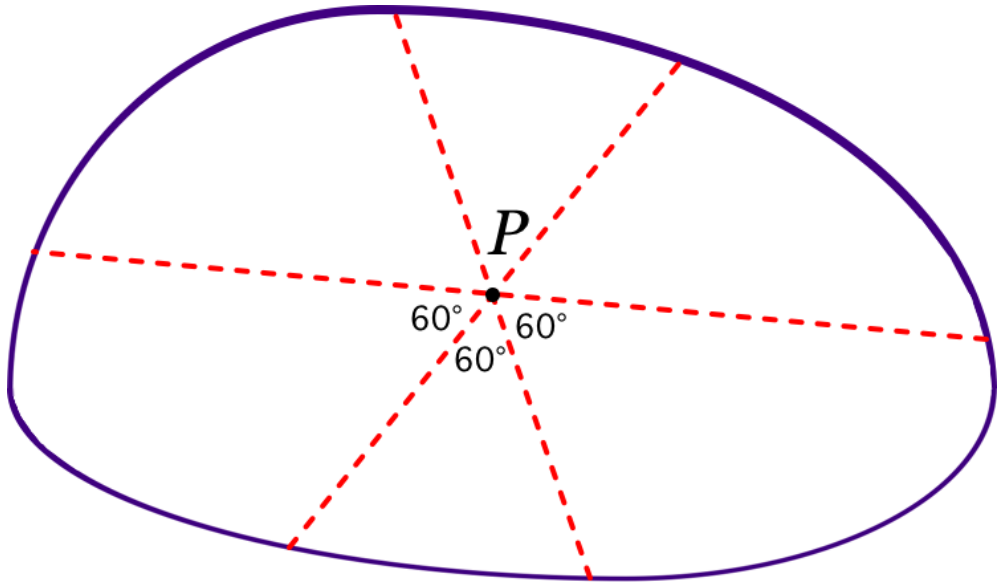
- True
- False



Grand Finale

Given any blob, it is always possible to locate an equiangular trisplit point.

- True
- False



Grand Finale

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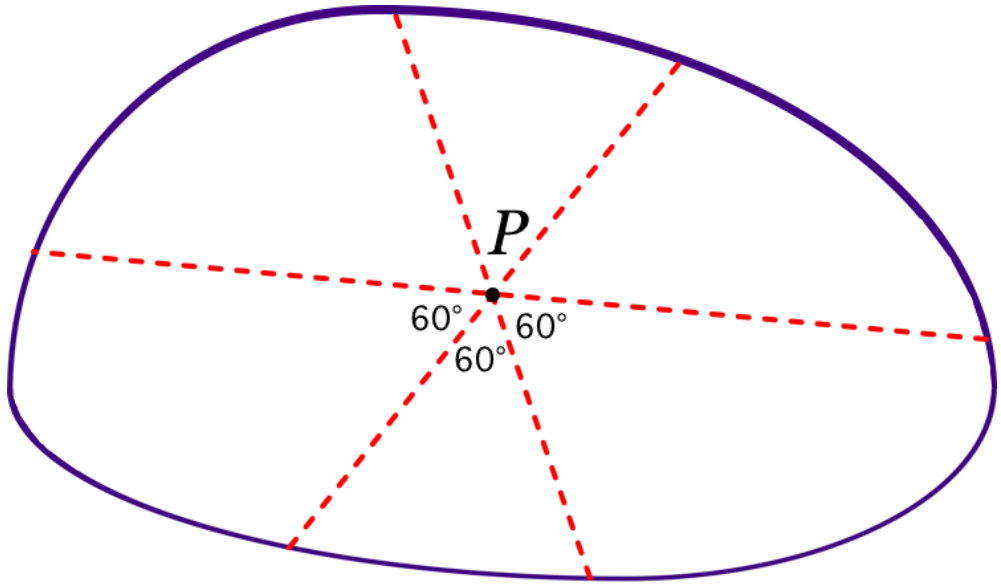
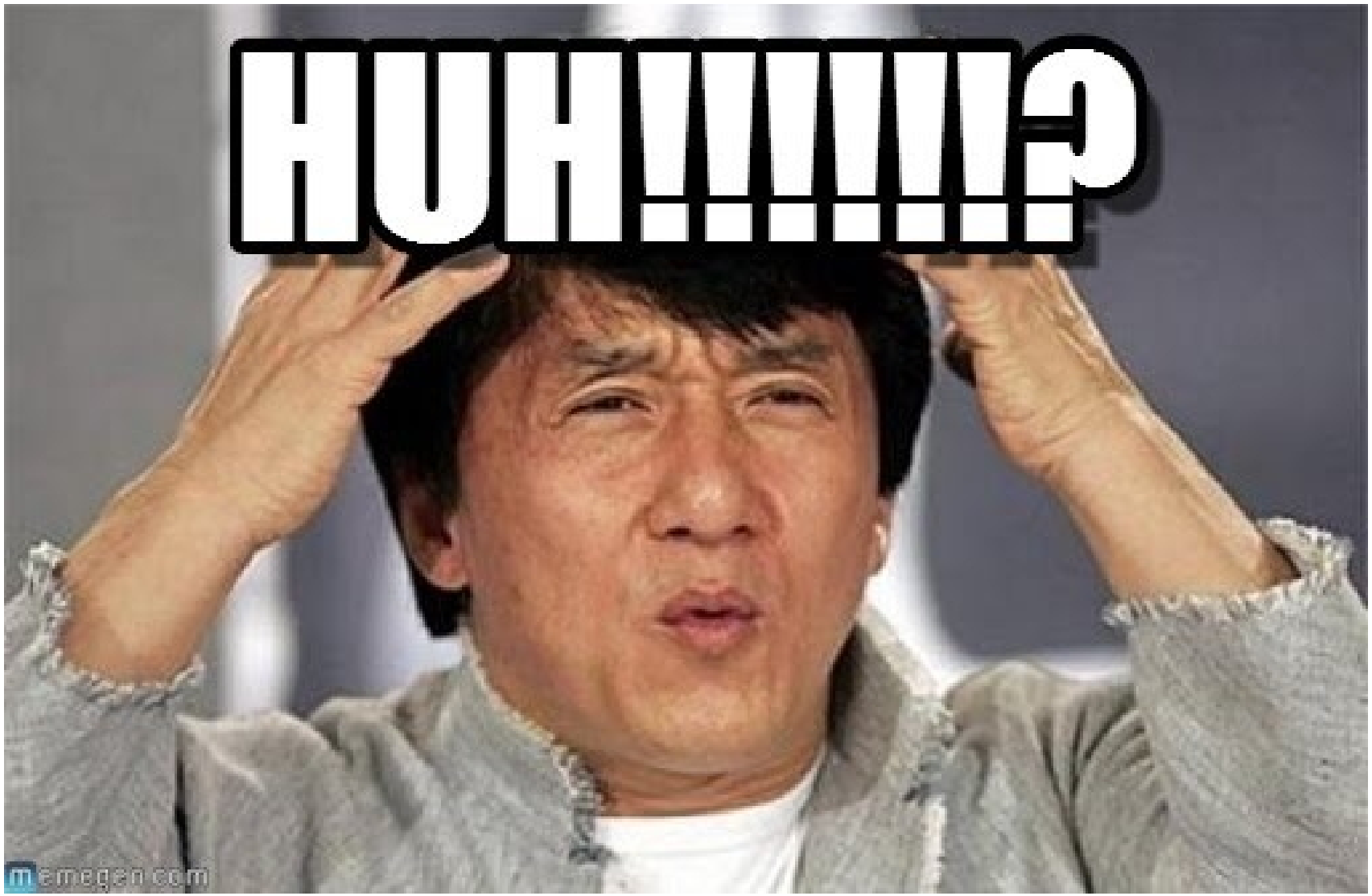
- TRUE ⟸
- FALSE



Too Much To Ask



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There's nothing to it, really.

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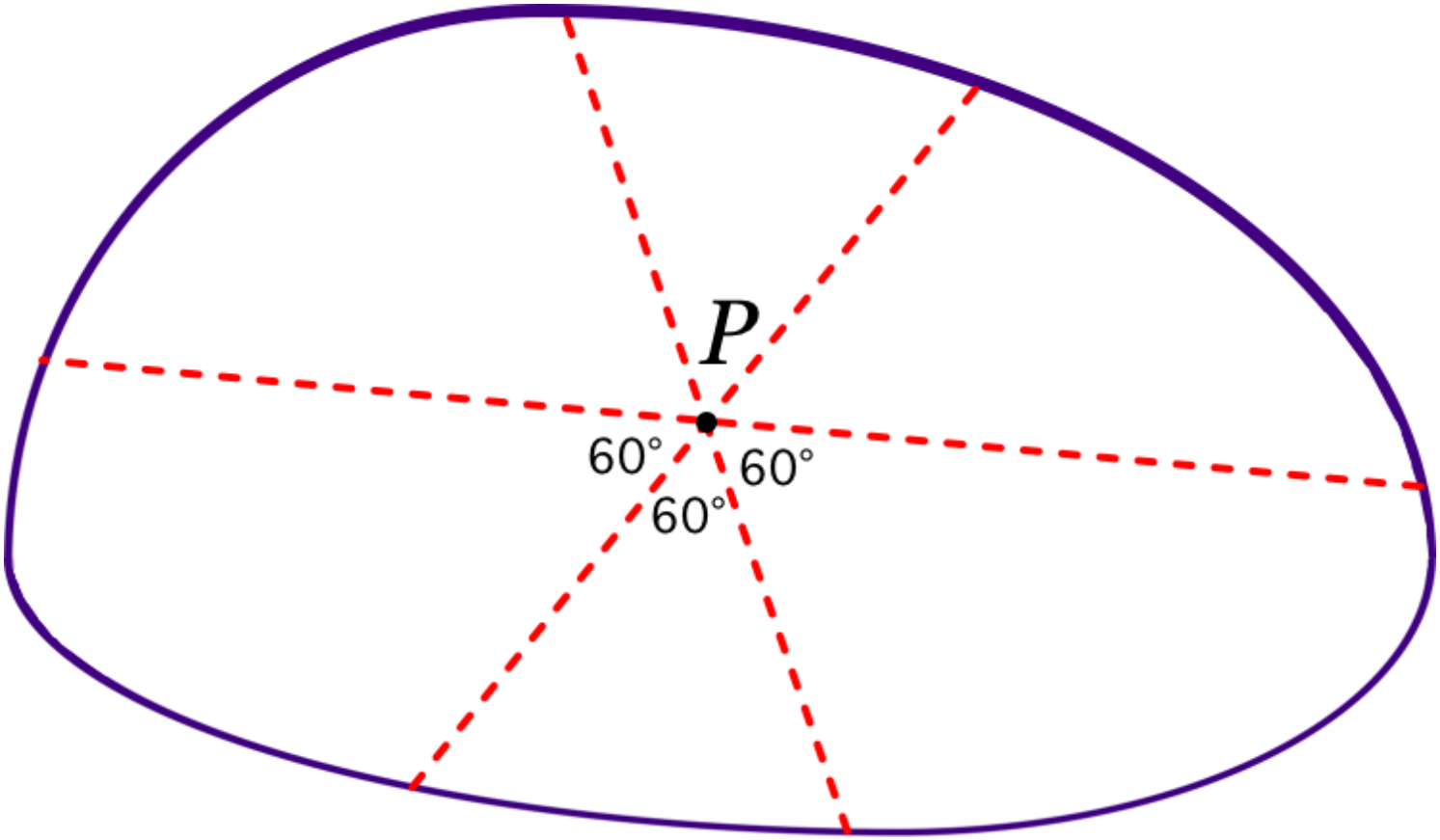
- Given any pair of directions θ_1 and θ_2, there is a unique bisplit point.
- Track the bisplit point for θ_1, $\theta_1 + 60°$, as θ_1 varies from $0°$ to $180°$.
- Consider the difference between the third pair of arms in direction $\theta_1 + 120°$.

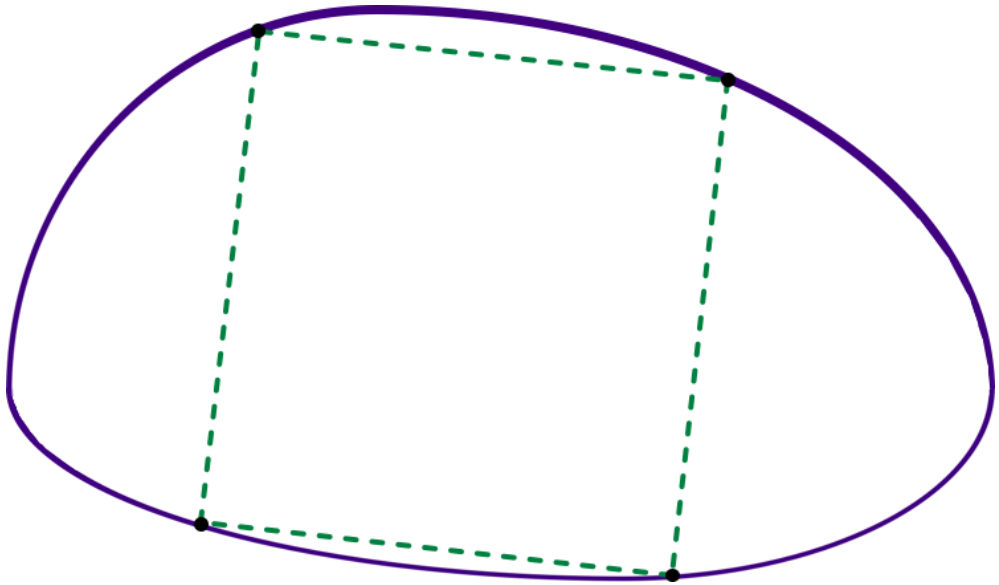
Grand Finale

And You're Done!



Bonus Features

Here's an extra challenge, just for fun. Show that every blob has an inscribed square.



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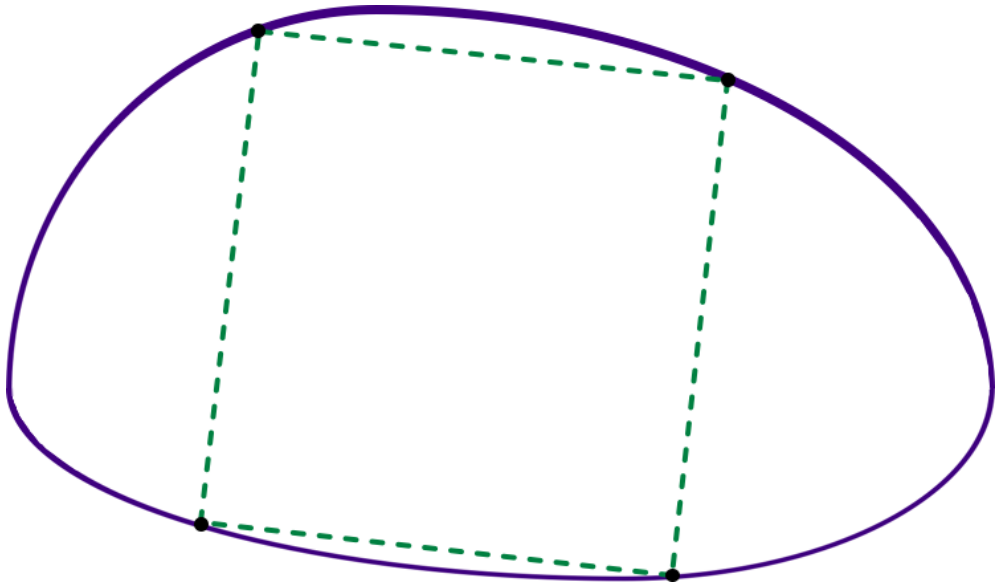

G O O D L U C K

Bloberrific

Way to hang in there. Time for

Amazing Digit Puzzle #123456

$$3 \cdot 4 = 12 \qquad 4 \cdot 13 = 52$$

What is the next product in this pattern?

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What is the next product in this pattern?

Well done! We have $3 \cdot 54 = 162$.

See You Tomorrow

Thanks for being such a great audience!